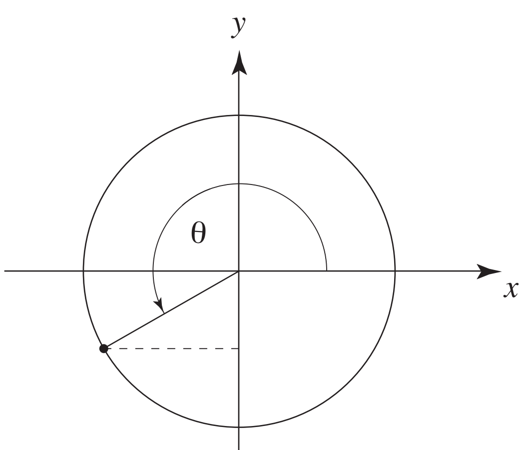
Exam B (Part I) **Name**

Please Note: If you wish full credit for your work, provide complete but efficient explanations, write clearly, and organize your work well. Suggestion: Do your initial thinking on scrap paper. Only thereafter proceed to the test making careful use of the space designated for the problem. Calculators can be used only in elementary arithmetic and trig mode. Only *lucid, well organized, and complete* solutions will receive full credit.

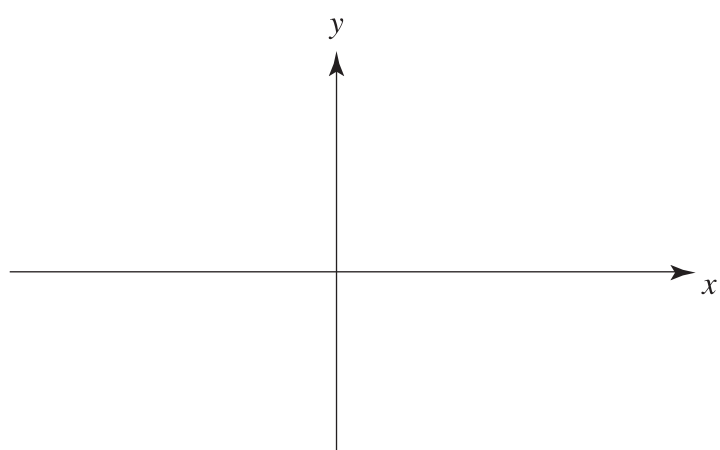
1. Draw a circle with diameter AB and center C. Let P be a point on the circle such that $\angle PCB = 30°$. Compute $\sin 15°$ and $\cos 15°$.

2. Consider an angle θ. Express $\sin(\theta + \frac{\pi}{4})$ in terms of $\sin\theta$ and $\cos\theta$.

3. Figure (a) below shows a circle of radius 1 centered at the origin and an angle θ. Provide estimates for $\sin\theta$ and $\cos\theta$. (Put your estimates into the box provided.)



(a)



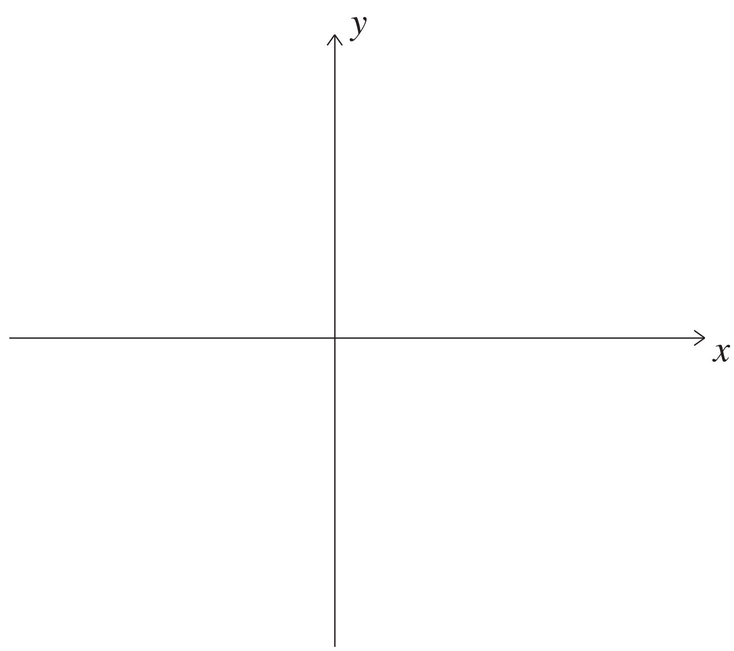
(b)

$\sin\theta \approx$ $\cos\theta \approx$

4. Using Leibniz's tangent method compute the slope of the tangent to the curve $y^2 = x^3 + 4$ at $P = (1, \sqrt{5})$. At which points does the graph have an horizontal tangent?

4. Consider the circle $x^2 + y^2 = 9$ and the parabola $y = \frac{1}{3}x^2 - 3$.

4a. Determine all the points of intersection of the graphs of the two equations. Sketch both of the graphs including all the points of intersection on the coordinate plane below.



4b. Sketch the region $R = \{(x, y) \mid -\sqrt{9 - x^2} \leq y \leq \frac{1}{3}x^2 - 3\}$ into your diagram.

4c. Determine the area of the region R by making use of Archimedes' Theorem.

5. Consider the function $f(x) = 16 - x^2$ with $-2 \leq x \leq 2$. Select the points $-2 \leq -1 \leq 0 \leq 1 \leq 2$ on the x-axis between -2 and 2.

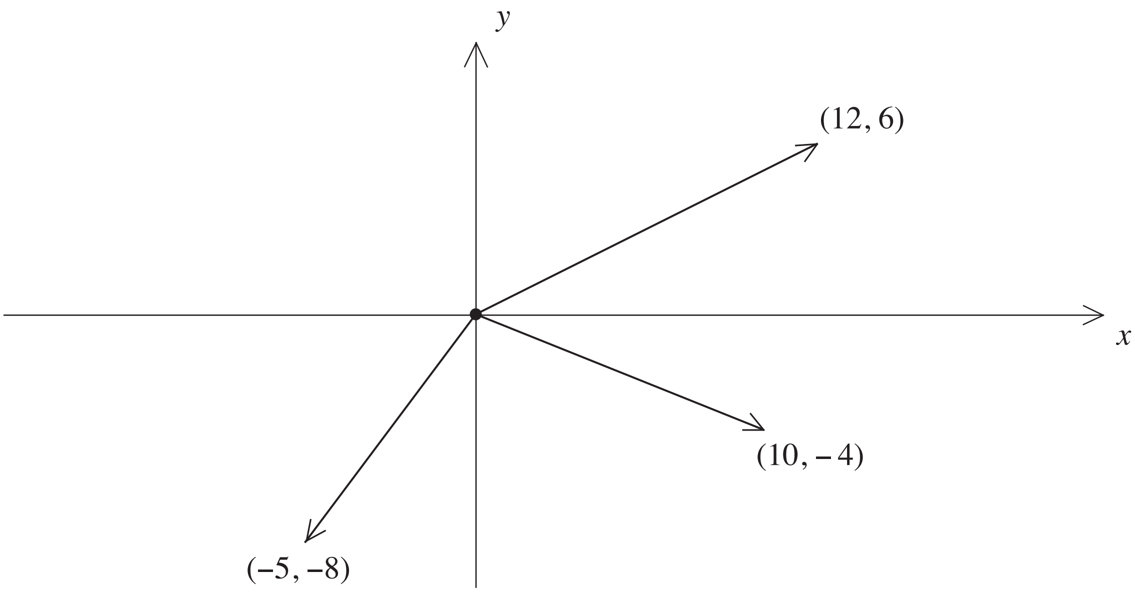
5a. Approximate the area under the graph of $f(x)$ over $-2 \leq x \leq 2$ by using Leibniz's strategy of Section 5.6.

5b. Apply Archimedes's theorem to find the area precisely.

5c. Use the fundamental theorem of calculus to find the area.

6. Use the power series expansion $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$ for x satisfying $|x| < 1$ to derive a power series expansion of $\frac{1}{1-2x^2}$. For what range of x is it valid. Then use this expansion to approximate $\int_0^{\frac{1}{3}} \frac{1}{1-2x^2}\, dx$ with two decimal place accuracy.

7. The three arrows in the diagram below are vectors representing forces. Each of the forces acts on a point at the origin of an xy-coordinate system. The endpoints of each of the vectors is indicated.



i. Determine the magnitudes of the horizontal and vertical components of each force.

ii. Determine the resultant of the three forces and place the arrow corresponding to it into the diagram.

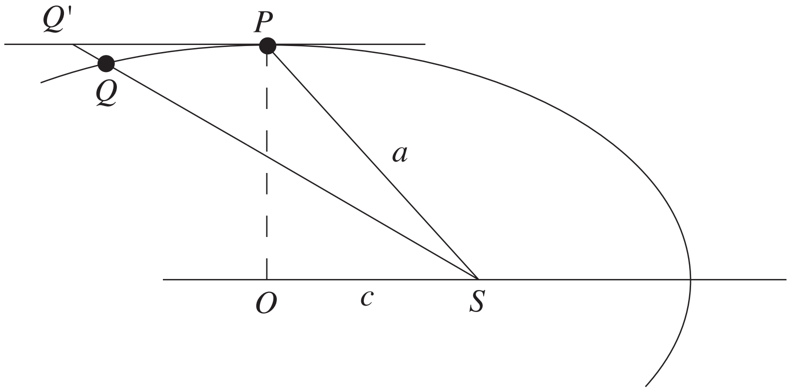
8. A point moves in the xy-plane. It starts at $(0, 0)$ at time $t = 0$. At any time $t \geq 0$ its position is given by $(x(t), y(t))$. It is known that at any time t its velocities in the x and y direction are 2 and $t^2 - 3$, respectively.

8a. Determine the functions $x(t)$ and $y(t)$ in precise terms. Where is the point at time $t = 1$?

8b. Determine an equation for the path of the point (i.e. express y as a function of x), sketch a graph of this function, discuss the shape of the path, and explain how the point moves.

8c. Suppose that the point has a mass of 1 unit and discuss the forces acting on it.

9. The figure below considers a planet in an elliptical orbit. The point O is the center of the ellipse. The semimajor axis of the ellipse is a and the period of the orbit is T. Consider the planet at position P at the "top" of its orbit and again at a time Δt later at Q. For a small Δt, PQ is approximately equal to PQ' and the area of the elliptical sector SPQ is approximately equal to the



area of the triangle SPQ'. Let $\Delta s = Q'P$ and make use of Kepler's constant $\kappa = \frac{ab\pi}{T}$ to approximate the average velocity $\frac{\Delta s}{\Delta t}$ of the planet from P to Q. Then push Δt to zero to show that the velocity of the planet at P is equal to $v_P = \frac{2\pi a}{T}$.

Some Formulas and Facts:
Area of circular sector equals: $\frac{1}{2}\theta r^2$
Archimedes's theorem: Area of parabolic section $= \frac{4}{3} \times$ Area of inscribed triangle.
$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$